# Delay-induced resonances in an optical system with feedback

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We study the influence of the delay time in the response of a delayed feedback system to external periodic driving. The nonlinear system we consider is a semiconductor laser with optical feedback operating in the low-frequency fluctuation regime. We numerically examine the consequences of varying the external cavity length of the system when a weak modulation is introduced through the laser's pump current. The harmonic modulation is seen to lead to a partial periodic entrainment of power dropouts, and the distribution of time intervals between the dropouts exhibits resonances with certain delay times. In other words, the response of the system to the external modulation is enhanced for particular values of the external cavity length. The same effect can be observed in the presence of noise, indicating that stochastic resonance can be enhanced or degraded depending on the feedback time.

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The control of the dynamics of nonlinear systems has been a field of great interest in recent years [1]. Carefully chosen small perturbations, for instance, can stabilize the otherwise unstable limit cycles embedded in a strange attractor, rendering an originally chaotic dynamics periodic [2]. In a more direct approach, parametric modulation has also been used to induce periodic behavior in chaotic systems, by entraining their dynamics to the external driving [3]. The efficiency of such an entrainment is an important issue. Here we consider the case of a chaotic system with delayed feedback, and show that the entrainment efficiency is substantially affected by the feedback time.

The nonlinear system that we study is a semiconductor laser with external optical feedback (see Fig. 1) [4]. This system has attracted much attention in the last decades, due in part to its potential application to optical communications. Semiconductor lasers are highly nonlinear, and in the presence of optical feedback from an external mirror they exhibit a rich variety of dynamical regimes when control parameters (basically the laser pumping intensity and the feedback strength) are carefully adjusted. One of their dynamical regimes of operation, probably the most studied one, is the low-frequency fluctuation regime (LFF) [5], in which the total output intensity of the laser turns off abruptly at irregular times, recovering gradually after a short time interval. A dynamical interpretation of this phenomenon can be obtained from the Lang-Kobayashi model [4], a system of delay-differential equations that describes the behavior of the emitted electric field and population inversion in the assumption of single-longitudinal-mode behavior and weak reflectivity of the external mirror (see below). According to this model, the fixed points of the system dynamics in the presence of feedback are pairs of external cavity modes and their corresponding antimodes (which correspond to constructive and destructive interference, respectively, between the intracavity and the reinjected light beams). Sano [6] showed that intensity dropouts are a consequence of the collision of the system trajectory with a saddle-type antimode.

Different schemes have been proposed to suppress and control these chaotic dropouts. A second external cavity, for instance, has been seen to stabilize the system and suppress the dropouts [7,8]. On the other hand, a harmonic modulation of the pump current has been used to entrain the otherwise irregular dropouts to the periodic driving, thereby eliminating the chaotic behavior of the system [9]. This procedure provides a good entrainment of the intensity dropouts at a wide range of modulation periods [10]. Further work has shown that entrainment to a weak periodic modulation can be enhanced by adding noise to the pump current [11,12], in an example of stochastic resonance. Finally, a relatively periodic response of the dropout events has been reported even in the absence of a harmonic modulation, provided noise is still added to the pump current [13,14] in a form of coherence resonance. In the latter two cases, both the intensity and the correlation time of the noise have been seen to be critical parameters for entrainment to happen [12,14].

In the present paper we are concerned about how the feedback time influences the entrainment of power dropouts in a semiconductor laser. To that end, we use the above-mentioned Lang-Kobayashi model to numerically study the distribution of time intervals between dropout events for varying lengths of the external cavity. As will be shown below, our results indicate that the response of the laser exhib-

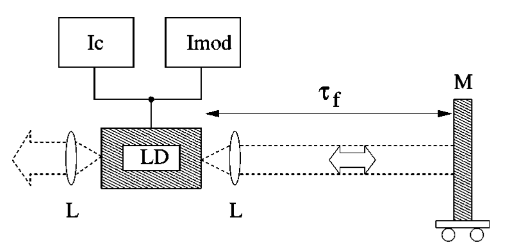


FIG. 1. Setup studied: LD is the laser diode, L are the collimating lenses, and M is an external mirror, whose distance to the laser can be modified.

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TABLE I. Semiconductor laser parameters used in the simulations.

| Description | Symbol | Value |
|---|---|---|
| Linewidth enhancement factor | $\alpha$ | 5.0 |
| Cavity loss coefficient | $\gamma$ | 0.158 $ps^{-1}$ |
| Carrier inverse lifetime | $\gamma_e$ | $6.00\times10^{-4}$ $ps^{-1}$ |
| dc injection current | $C_0$ | 1.02 |
| Saturation coefficient | s | $3.0\times10^{-7}$ |
| Spontaneous emission noise | $\beta$ | $0.5\times10^{-9}$ $ps^{-2}$ |
| Differential gain coefficient | $g$ | $2.79\times10^{-9}$ $ps^{-1}$ |
| Carrier number at transparency | $N_0$ | $1.51\times10^{8}$ |
| Feedback level | $\kappa$ | 0.02 $ps^{-1}$ |
| External roundtrip time | $\tau_f$ | variable |

its resonances with respect to the feedback time. These resonances are of special importance in the stochastic resonance phenomenon, which is enhanced if the external cavity length is accurately adjusted. Figure 1 depicts a scheme of the specific setup studied.

The model used in the numerical simulations consists of a couple of rate equations describing the evolution of the slowly varying amplitude of the complex electric field $E(t)$ and the carrier number $N(t)$ [4]:

$$\frac{dE}{dt}=\frac{1+i\alpha}{2}[G(E,N)-\gamma]E(t)+\kappa e^{-i\omega\tau_f}E(t-\tau_f)+\sqrt{2\beta N}\zeta(t), \tag{1}$$

$$\frac{dN}{dt}=\gamma_e[C(t)N_{\rm th}-N(t)]-G(E,N)|E(t)|^2. \tag{2}$$

Here $\gamma$ and $\gamma_e$ are the inverse lifetimes of photons and carriers, respectively, $C(t)$ represents the pump current applied to the laser, $\omega$ is the emitting frequency of the solitary laser (without feedback), and $\alpha$ is the linewidth enhancement factor, which couples the amplitude and the phase of the electric field. The second term in Eq. (1) corresponds to the feedback, with $\kappa$ representing the feedback strength and $\tau_f$ the feedback roundtrip time. The last term of Eq. (1) is an internal noise that stands for spontaneous emission fluctuations, where $\beta$ measures the noise strength and $\zeta(t)$ is a Gaussian white noise with zero mean and unity intensity. The material gain function $G(E,N)$ is given by

$$G(E,N)=\frac{g[N(t)-N_0]}{1+s|E(t)|^2}, \tag{3}$$

where $g$ is the differential gain coefficient, $N_0$ is the carrier number at transparency and $s$ is the saturation coefficient. The threshold carrier number $N_{\rm th}$ in Eq. (2) is given by $N_{\rm th}=\gamma/g+N_0$.

For constant pump current $C(t)=C_0$ and the parameters of Table I, the laser operates in the LFF regime, characterized by irregularly spaced and sudden drops in power, accompanied by abrupt increases in the accumulated phase along the external cavity, $\eta(t)=\phi(t)-\phi(t-\tau_f)$, with $E(t)=\sqrt{I(t)}\exp[i\phi(t)]$ [see Ref. [14] and Fig. 2(a)]. If a periodic modulation is added to the pump current, i.e., $C(t)=C_0+A_{\rm mod}\sin(\omega_{\rm mod}t)$, the dropouts can be entrained to the external periodic driving for high enough modulation amplitude $A_{\rm mod}$, as shown in Ref. [9]. In that work it was observed that, for a given delay time, entrainment can be obtained for a certain range of modulation frequencies $\omega_{\rm mod}$, and it was conjectured that this entrainment is optimal when the modulation frequency is close to the frequency difference between an external cavity mode and its adjacent antimode [9]. If that is the case, the relation between the external cavity frequency and the modulation frequency can be expected to play an important role in the response of the system to periodic driving. Since the length of the external cavity determines its resonance frequency spectrum, the feedback time $\tau_f$ should influence the entrainment substantially. In order to examine this point, we modify the external roundtrip time while keeping the modulation frequency constant. With this aim, we set the dc pump current to $C_0=1.02$ (the threshold for lasing action in these units is $C_0=1$), and introduce a sinusoidal modulation of amplitude $A_{\rm mod}=0.04$ and period $T_{\rm mod}=70$ ns. Figure 2 shows the phase difference $\eta(t)$ for three different delay times. For the modulation parameters chosen, the dropouts are perfectly entrained to the driving signal for $\tau_f=2.8$ ns [Fig. 2(b)]. However, the entrainment is lost when the delay time is decreased [Fig. 2(a)], and changes qualitatively when it is increased [Fig. 2(c)]. Hence, the results indicate that an entrainment of the intensity dropouts at the modulation frequency does not occur for all delay times, due to the interplay between the modulation frequency and the external cavity frequency highlighted in Ref. [9].

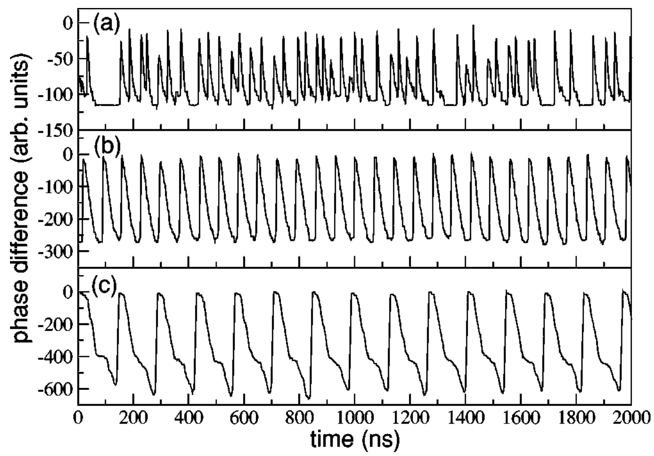


FIG. 2. External cavity phase difference $\eta(t)$ of a modulated laser with optical feedback in the LFF regime for three different values of the delay time: (a) $\tau_f=1$ ns, (b) $\tau_f=2.8$ ns, and (c) $\tau_f=8.4$ ns. The modulation period is $T_{\rm mod}=70$ ns in all cases, with a fixed amplitude of $A_{\rm mod}=0.04$. The rest of the laser parameters are those of Table I.

In order to examine the effect of the delay time $\tau_f$ systematically on the statistical distribution of the power dropouts, we plot, as a solid line in Fig. 3(a), the normalized standard deviation of the time intervals between consecutive dropouts for increasing $\tau_f$ and the same modulation ampli-

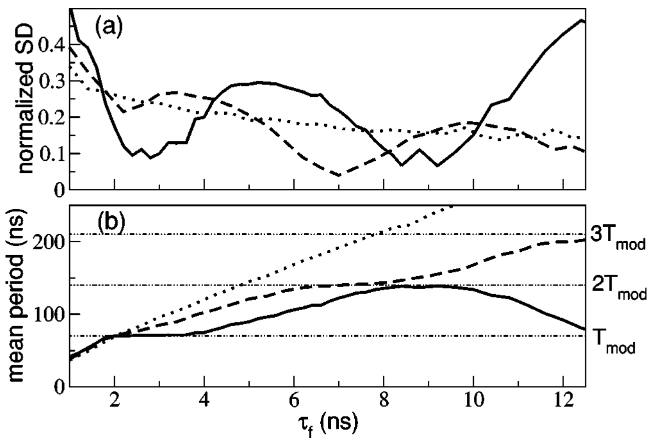


FIG. 3. Normalized standard deviation (a) and mean period (b) of the time interval between dropouts for three different amplitude modulations: $A_{\mathrm{mod}}=0$ (dotted line), $A_{\mathrm{mod}}=0.02$ (dashed line), and $A_{\mathrm{mod}}=0.04$ (solid line). In (b) horizontal guiding lines indicate the multiples of the modulation period $T_{\mathrm{mod}}$.

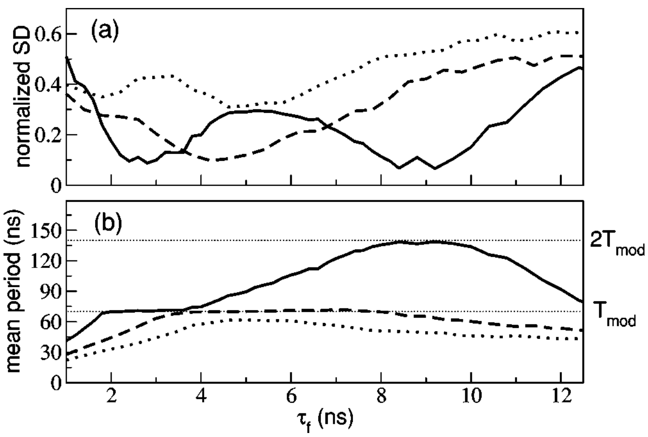


FIG. 4. Normalized standard deviation (a) and mean period (b) of the time interval between dropouts for three different noise amplitudes: $\sigma=0.00$ (solid line), $\sigma=0.04$ (dashed line), and $\sigma=0.06$ (dotted line). In (b) horizontal guiding lines indicate the multiples of the modulation period $T_{\mathrm{mod}}$.

tude $A_{\mathrm{mod}}=0.04$ as in Fig. 2. Two well defined resonances can be observed, characterized by two clearcut minima of the standard deviation for given values of the delay time. These resonances, however, are not a straightforward consequence of the relationship between the modulation frequency and the external cavity frequency. If that were the case, the resonances would occur at the same values of $\tau_f$ irrespective of the modulation amplitude (provided $T_{\mathrm{mod}}$ remains unchanged), since the distance between modes and adjacent antimodes depends only on $\tau_f$, not on $A_{\mathrm{mod}}$. But a comparison between the solid and dashed lines of Fig. 3(a) shows that the resonance minima shift to lower values of $\tau_f$ as the modulation amplitude decreases.

In order to clarify this fact, in Fig. 3(b) we plot the mean value of the interdropout intervals (to be called "mean period" in what follows) corresponding to the cases shown in Fig. 3(a). The results clearly show that when the resonances happen, the mean period locks to the modulation period $T_{\mathrm{mod}}$ or a multiple of it. This behavior clearly contrasts with the situation when no modulation is applied (dotted line in Fig. 3). In that case, the mean period increases monotonously with the delay time, and the normalized standard deviation depends only weakly (and also monotonically) on it.

The increase of the mean dropout period with feedback time for no (or small) modulation amplitude can be related to the fact that the number of external cavity modes increases with the delay time. For large modulation amplitudes, on the other hand, the periodic driving takes over the dynamics of the system, forcing the intensity to drop before its natural dropout period. The shift in the resonances with $A_{\mathrm{mod}}$ can then be understood from the fact that the modulation alters the dependence of the mean period on the delay time in such a way that locking occurs at different values of $\tau_f$ for different modulation amplitudes. As $A_{\mathrm{mod}}$ increases, the width of the locking regions increases, until, for a large enough modulation amplitude, an entrainment of the interval between dropouts is observed for all delay times (results not shown).

The previous results show that the feedback time must be accurately selected in order to enhance entrainment to a harmonic modulation. A natural question that arises from these results is how the feedback time affects the phenomenon of stochastic resonance (SR) in a semiconductor laser in the LFF regime [11,12]. With the aim of answering this question, we introduce in the system a time-correlated external noise $\xi(t)$ through the pumping current of the laser, superimposed onto the weak periodic signal, i.e., $C(t)=C_0+\xi(t)+A_{\mathrm{mod}}\sin(\omega_{\mathrm{mod}})$, where $\xi(t)$ is a Gaussian Ornstein-Uhlenbeck noise with zero mean and correlation:

$$\langle \xi(t)\xi(t')\rangle = \frac{D}{\tau_c} e^{-|t-t'|/\tau_c}. \qquad (4)$$

The external noise is characterized by its intensity $D$ and its correlation time $\tau_c$. The variance of the noise is given by $D/\tau_c$, so that its amplitude is $\sigma=\sqrt{D/\tau_c}$. A time-correlated noise is chosen due to the fast dynamics of this system ($\sim$ tens of picoseconds) [15], which makes the consideration of a white electronic noise an unrealistic assumption [14].

Earlier work has shown that noise of the type given in Eq. (4) can play the role of a modulation, in terms of enhancing entrainment [12]. We can thus expect that the delay-induced resonances described will also depend on the noise intensity. In order to test this conjecture, we set the modulation and laser parameters to match the situation presented in the solid curve of Fig. 3 and introduce noise into the system. Figure 4 shows the normalized standard deviation and the average of the time interval between dropouts for three different noise levels as a function of the delay time $\tau_f$. The results indicate that, as the noise intensity increases, the resonances shift to higher delay times, similarly to what happens in the deterministic case as the modulation amplitude increases. At variance with the deterministic case, however, for large noise intensities the standard deviation curve shifts upwards, as expected from a disordering effect of noise.

This dependence on the delay time is critical from the point of view of SR, which cannot be observed equally well

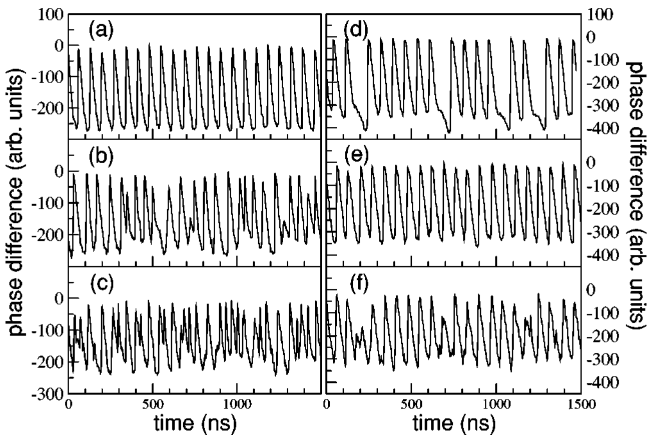


FIG. 5. External-cavity phase difference $\eta(t)$ for two different delay times: $\tau_f = 2.8$ ns (left), and $\tau_f = 4.3$ ns (right). The values of the noise amplitude are: $\sigma = 0.00$ (a,d); $\sigma = 0.025$ (b,e); and $\sigma = 0.041$ for (c,f).

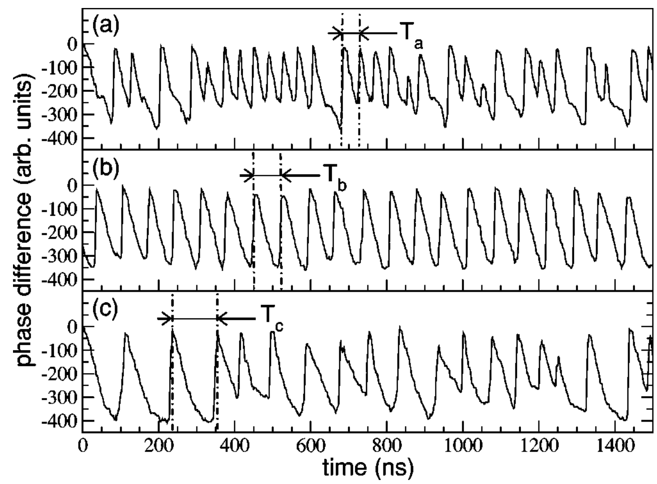


FIG. 6. External-cavity phase difference $\eta(t)$ for a fixed noise amplitude $\sigma = 0.025$, delay time $\tau_f = 4.3$ ns, and three different modulation periods: (a) $T_a = 40$ ns, (b) $T_b = 70$ ns, and (c) $T_c = 120$ ns.

for all delay times: as shown in Fig. 4(a), near the minima of the deterministic (solid) curve (a) noise can only degrade the regularity of the dropouts, and hence SR cannot be expected for these delay times. Far away from these minima, on the other hand, a small amount of noise will improve the quality of the entrainment (which will be degraded again for large enough noise), and hence SR will arise. The farther the operating point is from these minima, the more pronounced the SR effect will be. This fact is shown in Fig. 5, which plots the phase difference $\eta(t)$ between the emitted and reinjected fields for two different delay times and for three increasing noise intensities. As expected from the discussion above, SR is not observed at $\tau_f = 2.8$ ns, where increasing noise destabilizes the regular output of the laser [see Figs. 5(a)–5(c)]. For $\tau_f = 4.3$ ns, on the other hand, intermediate values of noise enhance the regularity of the laser output [Figs. 5(d)–5(f)], which is the typical feature of SR.

Our results indicate that the response to external modulation of certain types of delayed nonlinear systems can be optimized by tuning the magnitude of the delay time. In spite of the general interest of such a conclusion, we must remark that experimentally observing this phenomenon in the particular laser system studied in this paper is a challenging task, due mainly to the difficulty of reproducing the alignment of the external mirror and the amount of feedback for different cavity lengths. Therefore the optimization procedure reported here, if intended to improve the design and fabrication of an integrated device, should be carefully pursued at the very beginning of the design process. From the scientific point of view, since as mentioned above there is a clear relationship between the external cavity length and the modulation period, a parallel experiment can be proposed consisting in fixing the external cavity length and modifying the modulation period. Figure 6 shows the phase difference time traces obtained numerically in that case, when the pumping current is modulated with three different modulation periods but with the same amplitude ($A_{\mathrm{mod}} = 0.04$), and in the presence of external noise ($\sigma = 0.025$). The results indicate that, for these particular conditions, the entrainment is only achieved for intermediate values of modulation period [Fig. 6(b)], i.e., a resonant effect is also observed in this case.

Many types of real-life nonlinear systems are subjected to the joint influence of delayed feedback, external driving, and noise. In this work, we have numerically analyzed the interplay between these three factors in a well-controlled nonlinear device, namely, a semiconductor laser with optical feedback modulated by a weak pump current. Our results show that resonances with the delay time (i.e., with the external cavity length) exist, and that their location shifts with the modulation amplitude. This latter fact indicates that the resonances are not a simple consequence of the matching between the modulation frequency and the beating frequency of adjacent external cavity modes. Instead, we show that the resonances correspond to locking of the mean period between dropouts to multiples of the modulation period. We have also studied how this effect influences the role of noise in this system, showing that by adjusting the delay time one can either enhance or degrade the phenomenon of stochastic resonance. It would be interesting to establish whether a similar effect of the delay time exists in other pulsating systems exhibiting stochastic resonance, such as for instance neural systems.

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